

# Thermal Sinusoidal Vibration and Transient Response of Magnetostrictive Functionally Graded Material Plates without Shear Effects

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## Abstract

The study of laminated magnetostrictive functionally graded material (FGM) plate without shear deformation under thermal sinusoidal vibration and transient response is calculated by using the generalized differential quadrature (GDQ) method. In the thermoelastic stress-strain relations that containing a power-law function of a two-material FGM plate, the linear temperature rise and the magnetostrictive coupling terms with velocity feedback control. Four edges of rectangular laminated Terfenol-D FGM plate with simply supported boundary conditions are considered. The suitable product value of coil constant and control gain can be used to reduce the amplitude of center displacement into a smaller value.

## Keywords

*Magnetostrictive; FGM; Shear Deformation; Thermal Vibration; GDQ; Velocity Feedback Control*

## Introduction

Typical functionally graded material (FGM) is usually made of different phases constituent materials, for example, the ceramic and metal used in the engine combustion chamber to withstand ultra-high-temperature and to reduce the stress singularities, respectively. Chi and Chung (2006) presented the mechanical analysis of FGM plates subjected to transverse load. There were several studies in the transverse displacements for the laminated plate including the shear deformation effect. Amabili and Farhadi (2009) made the research of the shear deformable versus classical theories for nonlinear vibrations of rectangular isotropic and laminated composite plates. The over-prediction of natural frequencies in the solution without shear deformation was found. Ray and Shivakumar (2009) analyzed the effect of shear deformation on the

piezoelectric fiber-reinforced composite plate by using the finite element method (FEM). Nguyen et al. (2008) obtained the static numerical results for the FGM plate with the effect of shear deformation.

Magnetostrictive material Terfenol-D has the magneto-electric coupling property under the action of magnetism and mechanism. Hong (2009) used the computational generalized differential quadrature (GDQ) method to study the transient responses of magnetostrictive plates under thermal vibration. Thermal stresses and center displacement with and without shear effect were calculated in the thin and thick plate, respectively. Ramirez et al. (2006) obtained the free vibration solution for magneto-electro-elastic laminates through the Ritz approach. Lee and Reddy (2005) analyzed the non-linear response of laminated plate of magnetostrictive material under thermo-mechanical loading by using the FEM. Lee et al. (2004) obtained the transient vibration values of displacement for the Terfenol-D plate including the effect of shear deformation by using the FEM. Hong (2007) used the GDQ method to make the thermal vibration study for the Terfenol-D magnetostrictive laminated plate with the first-order shear deformation. Hong (2012) used the GDQ method to make the Terfenol-D FGM plate analyses under rapid heating induced vibration with the shear deformation effect. It is interesting to study thermal vibration of the transverse center displacement and thermal stress in the Terfenol-D FGM plate without the shear deformation effect by using the GDQ method.

## Formulation

### FGM

Most materials of FGM can be used in the environment of higher temperature and can be

expressed in series form as follows by Chi and Chung (2006).

$$P_{fgm} = \sum_{i=1}^{n_m} P_i V_i. \quad (1)$$

where  $P_{fgm}$  is the material properties of FGM,  $n_m$  is the number of materials mixed to form the FGM,  $V_i$  is the volume fractions, and  $\sum_{i=1}^{n_m} V_i = 1$  for all constituent materials,  $P_i$  is the individual constituent material properties, usually with the form as follows.

$$P_i = P_0 (P_{-1} T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3). \quad (2)$$

in which  $P_0, P_{-1}, P_1, P_2$  and  $P_3$  are the temperature coefficients,  $T$  is the temperature of environment.

We denote the parameters for a two-material ( $n_m = 2$ ) FGM plate as follows.  $a$  and  $b$  is the length in the  $x$ ,  $y$  direction of the plate,  $h^*$  is the total thickness of magnetostrictive layer and FGM plate,  $h_3$  is the thickness of magnetostrictive layer,  $h_1$  and  $h_2$  are the thickness of FGM material 1 and FGM material 2, respectively,  $p_1$  and  $p_2$  are the in-plane distributed forces,  $q$  is the applied pressure load. The sum of volume fractions is in the form:  $V_1 + V_2 = 1$ , the variation form of  $V_2$  used in the power-law function is  $V_2 = (\frac{z+h/2}{h})^{R_n}$ , where  $z$  is the thickness coordinate,  $h$  is the thickness of FGM plate,  $R_n$  is the power-law index. And the material properties for equation (1) can be assumed for the simple calculation and expressed as follows by Hong (2012).

$$E_{fgm} = (E_2 - E_1) \left( \frac{z+h/2}{h} \right)^{R_n} + E_1, \quad (3a)$$

$$\nu_{fgm} = (\nu_2 + \nu_1) / 2, \quad (3b)$$

$$\rho_{fgm} = (\rho_2 + \rho_1) / 2, \quad (3c)$$

$$\alpha_{fgm} = (\alpha_2 + \alpha_1) / 2, \quad (3d)$$

shear stresses and shear strains in the laminate

$$\kappa_{fgm} = (\kappa_2 + \kappa_1) / 2. \quad (3e)$$

where  $E$  is the Young's modulus,  $\nu$  is the Poisson's ratio,  $\rho$  is the density,  $\alpha$  is the thermal expansion coefficients,  $\kappa$  is the thermal conductivity, the subscript  $fgm$  represents the FGM plate, the subscripts 1 and 2 represent the constituent material 1 and 2, respectively. The property terms  $E_1$   $E_2$   $\nu_1$   $\nu_2$   $\rho_1$   $\rho_2$   $\alpha_1$   $\alpha_2$   $\kappa_1$   $\kappa_2$  are expressed corresponding to term  $P_i$  in equation (2).

### GDQ Method

The GDQ method approximates the derivative of function, and the first-order and the second-order derivatives of function  $f^*(x, y)$  at coordinates  $(x_i, y_j)$  of grid point  $(i, j)$  can be discretized in series forms by Shu and Du (1997) and rewritten as follows:

$$\frac{\partial f^*}{\partial x} \Big|_{i,j} \approx \sum_{l=1}^N A_{i,l}^{(1)} f_{l,j}^*, \quad (4a)$$

$$\frac{\partial f^*}{\partial y} \Big|_{i,j} \approx \sum_{m=1}^M B_{j,m}^{(1)} f_{i,m}^*, \quad (4b)$$

$$\frac{\partial^2 f^*}{\partial x^2} \Big|_{i,j} \approx \sum_{l=1}^N A_{i,l}^{(2)} f_{l,j}^*, \quad (4c)$$

$$\frac{\partial^2 f^*}{\partial y^2} \Big|_{i,j} \approx \sum_{m=1}^M B_{j,m}^{(2)} f_{i,m}^*, \quad (4d)$$

$$\frac{\partial^2 f^*}{\partial x \partial y} \Big|_{i,j} \approx \sum_{l=1}^N A_{i,l}^{(1)} \sum_{m=1}^M B_{j,m}^{(1)} f_{l,m}^*. \quad (4e)$$

where  $A_{i,j}^{(m)}$  and  $B_{i,j}^{(m)}$  denote the weighting coefficients for the  $m$ th-order derivative of the function  $f^*(x, y)$  with respect to  $x$  and  $y$  directions.

### Thermo Elastic Stress-Strain Relations with Magnetostrictive Effect

We consider a rectangular laminated magnetostrictive FGM plate of the length  $a, b$  in the  $x$ ,  $y$  direction, respectively, under uniformly distributed load and thermal effect. There are no without shear effect assumption. The plane stresses

in a laminated FGM plate with magnetostrictive effect for the  $k^{th}$  layer are in the following equations by Lee and Reddy (2005):

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}_{(k)} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_{(k)} \begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \varepsilon_{xy} - \alpha_{xy} \Delta T \end{Bmatrix}_{(k)} - \begin{bmatrix} 0 & 0 & \tilde{e}_{31} \\ 0 & 0 & \tilde{e}_{32} \\ 0 & 0 & \tilde{e}_{36} \end{bmatrix}_{(k)} \begin{Bmatrix} 0 \\ 0 \\ \tilde{H}_z \end{Bmatrix}_{(k)} \quad (5)$$

where  $\alpha_x$  and  $\alpha_y$  are the coefficients of thermal expansion,  $\alpha_{xy}$  is the coefficient of thermal shear,  $\bar{Q}_{ij}$  is the stiffness of magnetostrictive FGM plate, the simpler forms of  $\bar{Q}_{ij}$  for FGM are given as follows.

$$\bar{Q}_{11} = \bar{Q}_{22} = \frac{E_{fgm}}{1 - \nu_{fgm}^2}, \quad (6a)$$

$$\bar{Q}_{12} = \frac{\nu_{fgm} E_{fgm}}{1 - \nu_{fgm}^2}, \quad (6b)$$

$$\bar{Q}_{44} = \bar{Q}_{55} = \bar{Q}_{66} = \frac{E_{fgm}}{2(1 + \nu_{fgm})}, \quad (6c)$$

$$\begin{bmatrix} A_{11} & 2A_{16} & A_{66} & A_{16} & A_{12} + A_{66} & A_{26} & 0 & 0 & 0 \\ A_{16} & A_{12} + A_{66} & A_{26} & A_{66} & 2A_{26} & A_{22} & 0 & 0 & 0 \\ B_{11} + B_{16} & 2B_{16} + B_{12} + B_{66} & B_{66} + B_{26} & B_{16} + B_{66} & B_{12} + B_{66} + 2B_{26} & B_{26} + B_{22} & 0 & 0 & 0 \end{bmatrix}$$

$$\left\{ \frac{\partial^2 u^0}{\partial x^2}, \frac{\partial^2 u^0}{\partial x \partial y}, \frac{\partial^2 u^0}{\partial y^2}, \frac{\partial^2 v^0}{\partial x^2}, \frac{\partial^2 v^0}{\partial x \partial y}, \frac{\partial^2 v^0}{\partial y^2}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial x \partial y}, \frac{\partial^2 w}{\partial y^2} \right\}^t$$

$$= \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} + \rho \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 u^0}{\partial t^2} \\ \frac{\partial^2 v^0}{\partial t^2} \\ \frac{\partial^2 w}{\partial t^2} \end{Bmatrix} + H \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{Bmatrix} \frac{\partial^2 u^0}{\partial t^2} \\ \frac{\partial^2 v^0}{\partial t^2} \end{Bmatrix} \quad (7)$$

Where  $f_1, f_2, f_3$  are the expressions of thermal loads  $(\bar{N}, \bar{M})$ , mechanical loads  $(p_1, p_2, q)$  and magnetostrictive loads  $(\tilde{N}, \tilde{M})$ ,

$$\bar{Q}_{16} = \bar{Q}_{26} = \bar{Q}_{45} = 0. \quad (6d)$$

$\varepsilon_x, \varepsilon_y$  and  $\varepsilon_{xy}$  are the in-plane strains,

$\Delta T = T_0(x, y, t) + \frac{z}{h^*} T_1(x, y, t)$  is the temperature difference between the FGM plate and curing area,  $z$  is the coordinate in the thickness direction,  $h^*$  is the plate total thickness,  $\tilde{e}_{ij}$  is the transformed magnetostrictive coupling modulus,  $\tilde{H}_z$  is the magnetic field intensity, expressed in the following equation by Lee et al. (2004),  $\tilde{H}_z(x, y, t) = k_c \tilde{I}(x, y, t)$  with velocity feedback control  $\tilde{I}(x, y, t) = c(t) \frac{\partial w}{\partial t}$ , in which  $k_c$  is the coil constant,  $\tilde{I}(x, y, t)$  is the coil current,  $c(t)$  is the control gain.

#### Dynamic Equilibrium Differential Equations

Without shear deformation effect, the time dependent of tangential displacement equations is assumed in the forms:  $u = u^0(x, y, t)$ ,  $v = v^0(x, y, t)$  and the transverse displacement equation of the middle-plane is assumed in the form:  $w = w(x, y, t)$  in which  $t$  is time. The dynamic equilibrium differential equations in terms of displacements including the magnetostrictive loads are expressed in the following matrix forms by Hong (2009):

$$f_1 = \frac{\partial \bar{N}_x}{\partial x} + \frac{\partial \bar{N}_{xy}}{\partial y} + p_1 + \frac{\partial \tilde{N}_x}{\partial x} + \frac{\partial \tilde{N}_{xy}}{\partial y},$$

$$f_2 = \frac{\partial \bar{N}_{xy}}{\partial x} + \frac{\partial \bar{N}_y}{\partial y} + p_2 + \frac{\partial \tilde{N}_{xy}}{\partial x} + \frac{\partial \tilde{N}_y}{\partial y},$$

$$f_3 = \frac{\partial \bar{M}_x}{\partial x} + 2 \frac{\partial \bar{M}_{xy}}{\partial y} + \frac{\partial \bar{M}_y}{\partial y} + q + \frac{\partial \tilde{M}_x}{\partial x} + 2 \frac{\partial \tilde{M}_{xy}}{\partial y} + \frac{\partial \tilde{M}_y}{\partial y},$$

$$(\bar{N}_x, \bar{M}_x) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} (\bar{Q}_{11}\alpha_x + \bar{Q}_{12}\alpha_y + \bar{Q}_{16}\alpha_{xy})(T_0, z \frac{z}{h^*} T_1) dz,$$

$$(\bar{N}_y, \bar{M}_y) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} (\bar{Q}_{12}\alpha_x + \bar{Q}_{22}\alpha_y + \bar{Q}_{26}\alpha_{xy})(T_0, z \frac{z}{h^*} T_1) dz,$$

$$(\bar{N}_{xy}, \bar{M}_{xy}) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} (\bar{Q}_{16}\alpha_x + \bar{Q}_{26}\alpha_y + \bar{Q}_{66}\alpha_{xy})(T_0, z \frac{z}{h^*} T_1) dz,$$

$$(\tilde{N}_x, \tilde{M}_x) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} \tilde{e}_{31} H_z(1, z^2) dz,$$

$$(\tilde{N}_y, \tilde{M}_y) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} \tilde{e}_{32} H_z(1, z^2) dz,$$

$$(\tilde{N}_{xy}, \tilde{M}_{xy}) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} \tilde{e}_{36} H_z(1, z^2) dz,$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} \bar{Q}_{ij}(1, z, z^2) dz, \quad (i, j = 1, 2, 6),$$

$$(\rho, H) = \int_{-\frac{h^*}{2}}^{\frac{h^*}{2}} \rho_0(1, z) dz.$$

in which  $\rho_0$  is the density of ply,  $p_1$  and  $p_2$  are the in-plane distributed forces,  $q$  is the applied pressure load.

### Dynamic Discretized Equations

Without the shear deformation effect and under the following vibration of time sinusoidal displacement and temperature:

$$u = [u^0(x, y)] \sin(\omega_{mn} t), \quad (8a)$$

$$v = [v^0(x, y)] \sin(\omega_{mn} t), \quad (8b)$$

$$w = w(x, y) \sin(\omega_{mn} t), \quad (8c)$$

$$\Delta T = [T_0(x, y) + \frac{z}{h^*} T_1(x, y)] \sin(\gamma t). \quad (8d)$$

where the term  $\omega_{mn}$  is the natural frequency of plate,  $\gamma$  is the frequency of applied heat flux.

We apply the weighting coefficients of discretized equations (4) in the two-dimensional GDQ method to discrete the differential equations (7). And we use the following non-dimensional parameters under sinusoidal temperature

$$(T_0 = 0, T_1 = \bar{T}_1 \sin(\pi x/a) \sin(\pi y/b)).$$

$$X = x/a, \quad (9a)$$

$$Y = y/b, \quad (9b)$$

$$U = u^0/a, \quad (9c)$$

$$V = v^0/b, \quad (9d)$$

$$W = 10h^* w / (\alpha_x \bar{T}_1 a^2). \quad (9e)$$

Thus, we can obtain the dynamic discretized matrix equations as in the paper by Hong (2009).

### Some Numerical Results and Discussions

We like to consider the FGM plate consisting of two materials, of which the FGM material 1 is SUS304 (Stainless Steel), the FGM material 2 is  $Si_3N_4$  (Silicon Nitride). The temperature-dependent coefficients  $P_0, P_{-1}, P_1, P_2$  and  $P_3$  used to calculate material property terms, and  $E_1, E_2, \nu_1, \nu_2, \rho_1, \rho_2, \alpha_1, \alpha_2$  and  $\kappa_1, \kappa_2$  of these two typical constituent materials are listed in **Table 1** by Shariyat (2008). The upper surface magnetostrictive layer of the three-layer ( $0^\circ / 0^\circ / 0^\circ$ ) laminates FGM plate under four sides simply supported is considered, the superscript of  $m$  denotes magnetostrictive layer. The elastic modules, material conductivity and specific heat of the Terfenol-D magnetostrictive material are used the same value as in the paper by Hong (2007). We use the grid points for the GDQ computation as in the following coordinates:

$$x_i = 0.5[1 - \cos(\frac{i-1}{N-1}\pi)]a, i = 1, 2, \dots, N, \quad (10a)$$

$$y_j = 0.5[1 - \cos(\frac{j-1}{M-1}\pi)]b, j = 1, 2, \dots, M. \quad (10b)$$

The dynamic convergence results are obtained for center displacement amplitude  $w(a/2, b/2)$  without shear effects in the thermal vibration of sinusoidal temperature only ( $T_0 = 0$ ,  $\bar{T}_1 = 100^\circ K$ ,  $p_1 = p_2 = q = 0$ ) at time  $t = 6s$ , mode shape  $m = n = 1$ , with control gain value  $k_c c(t) = 0$ , aspect ratio  $a/b = 0.5, 1$  and  $2$ , side-to-thickness ratio  $a/h^* = 100, 50, 20, 10$  and  $5$ ,  $h^* = 1.2$  mm,  $h_3 = 1$  mm,  $h_1 = h_2 = 0.1$  mm,  $R_n = 1$ ,  $T = 300^\circ K$ . **Table 2** shows the  $w(a/2, b/2)$  (unit mm) in the grid point  $N \times M = 17 \times 17$ ,  $21 \times 21$ ,  $25 \times 25$ ,  $29 \times 29$  and  $33 \times 33$  of GDQ method for the laminated Terfenol-D FGM plate at time  $t = 6$  s. We find the  $N \times M = 33 \times 33$  grid point have the good  $w(a/2, b/2)$  convergence result and use further in the GDQ analyses of time responses for displacement and stress for  $a/h^* = 5, 20$  and  $100$ .

We obtain the lowest frequency  $\gamma$  of applied heat flux and vibration frequency  $\omega_{11}$  of Terfenol-D FGM plate ( $m = n = 1$ ), at time  $t = 0.001s, 1s, 2s, \dots$  and  $9s$ ,  $N \times M = 33 \times 33$ ,  $h^* = 1.2$  mm,  $h_3 = 1$  mm,  $h_1 = h_2 = 0.1$  mm,  $a/b = 1$ ,  $q = 0$ ,  $R_n = 1$ ,  $T = 653^\circ K$ ,  $\bar{T}_1 = 100^\circ K$ , as shown in **Table 3**. For the magnetic coil constant  $k_c$  effect on the  $w(a/2, b/2)$  (unit mm) of vibration under the constant gain value  $c(t) = 1$ , time  $t = 6$  s,  $a/h^* = 5$  without shear, we obtain the sketch of  $w(a/2, b/2)$  vs.  $k_c$  as shown in **Fig. 1**. The suitable product value of  $k_c$  and  $c(t)$  can be used to reduce the amplitude of  $w(a/2, b/2)$  into a smaller value near 0.00, there is an amplitude peak value at  $k_c c(t) = 7.16E08$ , we find  $k_c c(t) = 1.56E09$  for  $a/h^* = 5$  thick plate can be chosen as the best suitable product values.

Firstly, thermal sinusoidal vibration is investigated with time step equal to 0.1s, the suitable chosen product  $k_c c(t)$  of coil constant and controlled gain

values versus time  $t$  for  $R_n = 1$ , thick plate  $a/h^* = 5$  and thin plate  $a/h^* = 20, 100$  at  $T = 653^\circ K$ , as shown in Table 4.

**Fig. 2** shows the  $w(a/2, b/2)$  (unit mm) versus time  $t$  of GDQ method for the laminated Terfenol-D FGM plate  $a/h^* = 5, 20$  and  $100$ , respectively without shear effects. At time  $t = 0.001s$ , there is a great amplitude value of displacement with uncontrolled value ( $k_c c(t) = 0$ ),  $w(a/2, b/2) = -0.156429$  mm for thick plate  $a/h^* = 5$ ,  $w(a/2, b/2) = -4.31174$  mm for thin plate  $a/h^* = 20$ ,  $w(a/2, b/2) = -0.22975$  mm for thin plate  $a/h^* = 100$ . We find the amplitudes of  $w(a/2, b/2)$  with controlled  $k_c c(t)$  values are smaller than the amplitudes of  $w(a/2, b/2)$  with uncontrolled value ( $k_c c(t) = 0$ ), generally by using the GDQ method. We can use the suitable product values of controlled  $k_c c(t)$  to reduce the amplitude of  $w(a/2, b/2)$  into a smaller value near 0.00.

**Fig. 3** shows the time response of the dominated dimensional stress  $\sigma_x$  (unit GPa) at center position of upper surface  $Z = 0.5h^*$  with respective to time for the laminated Terfenol-D FGM plate  $a/h^* = 5, 20$  and  $100$ , respectively without shear effects  $\alpha_{xy} = 0$  and  $\sigma_{xy} = 0$ . We find the maximum response values of  $\sigma_x = -9.13E-04$  GPa for  $a/h^* = 5$  and  $20$ ,  $\sigma_x = -9.12E-04$  GPa for  $a/h^* = 100$  with controlled  $k_c c(t)$  case are almost equal to the response values of  $\sigma_x = -9.12E-04$  GPa with uncontrolled case ( $k_c c(t) = 0$ ), generally by using the GDQ method.

**Fig. 4** shows the compared non-dimensional  $W(X, b/2, 6)$  versus  $X$  of  $a/h^* = 5, 20$  and  $100$  without / with shear effects, for Terfenol-D FGM plate, at time  $t = 6s$ ,  $m = n = 1$ ,  $N \times M = 33 \times 33$  for the case of without shear and  $17 \times 17$  for the case of with shear,  $h^* = 1.2$  mm,  $h_3 = 1$  mm,  $h_1 = h_2 = 0.1$  mm,  $a/b = 1$ ,  $q = 0$ ,  $R_n = 1$ ,  $T = 653^\circ K$ ,  $\bar{T}_1 = 100^\circ K$ ,  $k_c c(t) = 0$ . In the shear effect case, we use the YNS first-order shear deformation theory for the time dependent of displacement field, the value for shear correction

coefficients  $k_\alpha = k_\beta = 5/6$  used in the dynamic equilibrium differential equations by Hong (2012). For the thick plate  $a/h^* = 5$ , the values of displacement  $W(X, b/2, 6)$  versus  $X$  without shear case are smaller than that with shear case. The maximum value  $W(X, b/2, 6) = 0.0821655$  occurs at  $X = 0.5$  of the with shear case, and  $W(X, b/2, 6) = -0.0325304$  occurs at  $X = 0.691342$  of the without shear case. For the thin plate  $a/h^* = 20$ , the values of displacement  $W(X, b/2, 6)$  versus  $X$  without shear case are greater than that with shear case. The maximum value  $W(X, b/2, 6) = -0.00133522$  occurs at  $X = 0.777785$  of the without shear case, meanwhile  $W(X, b/2, 6) = 0.00033884$  occurs at  $X = 0.5$  of the with shear case. For the thin plate  $a/h^* = 100$ , the values of displacement  $W(X, b/2, 6)$  versus  $X$  without shear case are greater than that with shear case. The maximum value  $W(X, b/2, 6) = 0.0000138781$  occurs at  $X = 0.0380602$  of the without shear case, while  $W(X, b/2, 6) = 0.000000541036$  occurs at  $X = 0.5$  of the with shear case. The values of displacement  $W(X, b/2, 6)$  are decreasing with  $a/h^*$  values increasing for square plate  $a/b = 1$  with and without shear deformation effects. The deflection of center position versus  $a/h^*$  at  $t = 6$  sec in Fig. 4d shows the deflection of thick plate  $a/h^* = 5$ , and the case of deflection value with shear effect is much greater than the case of without shear effect. The deflections with and without shear effects are almost in the same values for the thin plate  $a/h^* = 100$ .

Secondly, transient response is investigated with time step equal to 0.001s and use the fixed frequency  $\gamma = 523.599/s$  of applied heat flux. The suitable chosen product  $k_c c(t)$  of coil constant and controlled gain values versus time  $t$  for  $R_n = 1$ , thick plate  $a/h^* = 5$  and thin plate  $a/h^* = 100$  at  $T = 653^\circ K$ , as shown in Table 5. We find the more thin plate is easier to control the displacement with less values of  $k_c c(t)$ .

Fig. 5 shows the transient value  $w(a/2, b/2)$  (unit mm) versus time  $t$  of GDQ method for the laminated Terfenol-D FGM plate  $a/h^* = 5$  and 100, respectively without shear effects. At time  $t = 0.001s$ , there is a great amplitude value of displacement with

uncontrolled value ( $k_c c(t) = 0$ ), from  $w(a/2, b/2) = -0.156429$  mm converges to small value for thick plate  $a/h^* = 5$ , from  $w(a/2, b/2) = -0.22975$  mm converges to small value for thin plate  $a/h^* = 100$ . We find the amplitudes of  $w(a/2, b/2)$  with controlled  $k_c c(t)$  values are smaller than the amplitudes of  $w(a/2, b/2)$  with uncontrolled value ( $k_c c(t) = 0$ ), generally by using the GDQ method. We can use the suitable product values of controlled  $k_c c(t)$  to reduce the amplitude of  $w(a/2, b/2)$  into a smaller value near 0.00.

Fig. 6 shows the transient value of the dominated dimensional stress  $\sigma_x$  (unit GPa) at center position of upper surface  $Z = 0.5h^*$  versus time for the laminated Terfenol-D FGM plate  $a/h^* = 5$  and 100, respectively without shear effects  $\alpha_{xy} = 0$  and  $\sigma_{xy} = 0$ . We find the transient values  $\sigma_x$  oscillate between  $-9.12E-04$ GPa and  $9.12E-04$ GPa for  $a/h^* = 5$  and 100. The transient values  $\sigma_x$  with controlled  $k_c c(t)$  case are almost equal to the values with uncontrolled case ( $k_c c(t) = 0$ ), generally by using the GDQ method.

## Conclusions

The GDQ calculation provides a method to compute the controlled displacement and stress in the  $(0^\circ / 0^\circ / 0^\circ)$  ply Terfenol-D FGM plate subjected to thermal vibration and transient response of sinusoidal temperature without shear deformation effect. The computation provides the following results. (a) The suitable controlled product values of coil constant and control gain  $k_c c(t)$  can be used to reduce the amplitude of center displacement  $w(a/2, b/2)$  into a smaller value near zero. (b) The amplitudes of stresses  $\sigma_x$  of plates are almost in the same values under the two cases: with and without  $k_c c(t)$  values. (c) The values of non-dimensional displacement  $W(X, b/2, 6)$  are decreasing with  $a/h^*$  values increasing for square plate  $a/b = 1$  under the two cases: with and without shear deformation effects. (d) The deflection values of center position versus  $a/h^*$  at  $t = 6$  sec are investigated, which found that the value with

shear effect is much greater than the case of without shear effect for the thick plate  $a/h^* = 5$ , as well the deflections with and without shear effects are almost in the same values for the thin plate  $a/h^* = 100$ .

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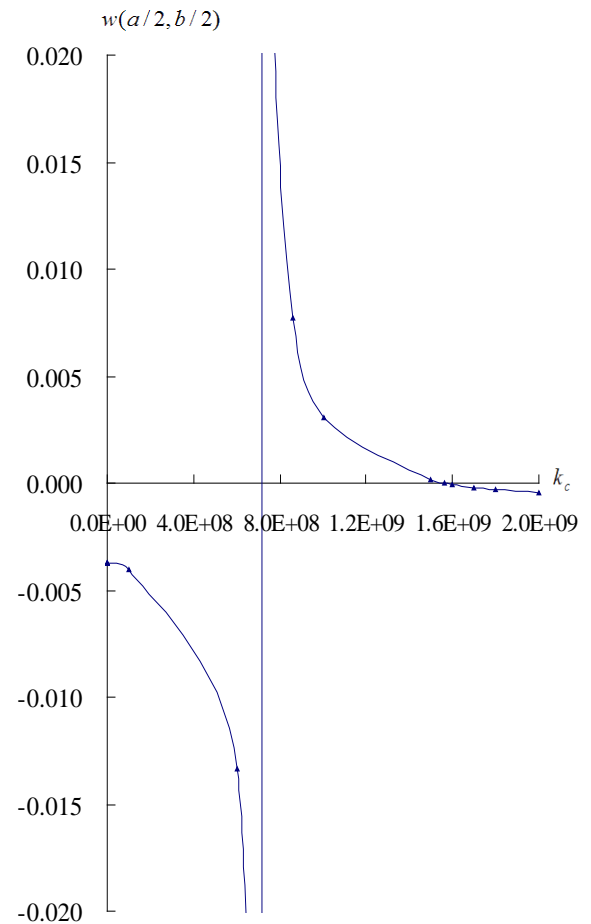


FIG. 1  $w(a/2, b/2)$  vs.  $k_c$  for  $a/h^* = 5$ ,  $c(t)=1$   
WITHOUT SHEAR

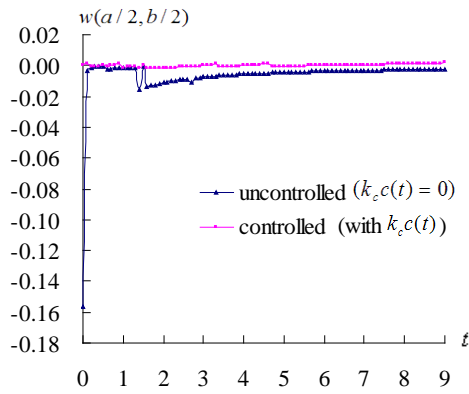


FIG. 2a  $w(a/2, b/2)$  vs.  $t$  for  $a/h^* = 5$

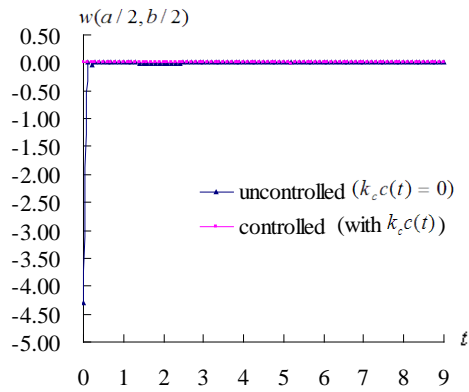


FIG. 2b  $w(a/2, b/2)$  vs.  $t$  for  $a/h^* = 20$

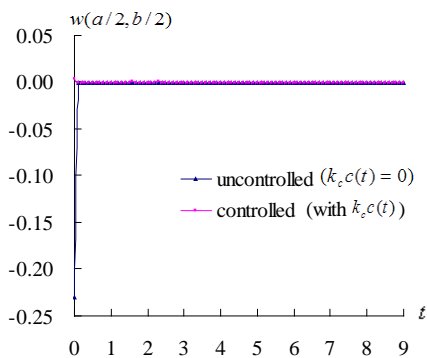


FIG. 2c  $w(a/2, b/2)$  vs.  $t$  for  $a/h^* = 100$

FIG. 2  $w(a/2, b/2)$  vs.  $t$  for  $a/h^* = 5, 20$  AND  $100$  WITHOUT SHEAR

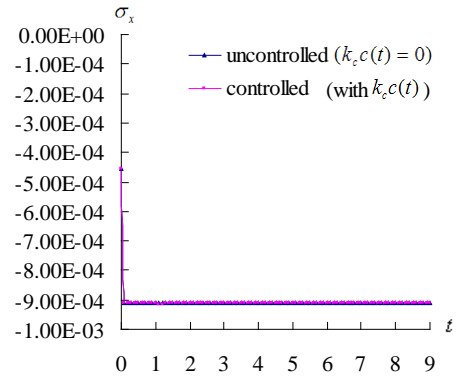


FIG. 3a  $\sigma_x$  vs.  $t$  for  $a/h^* = 5$

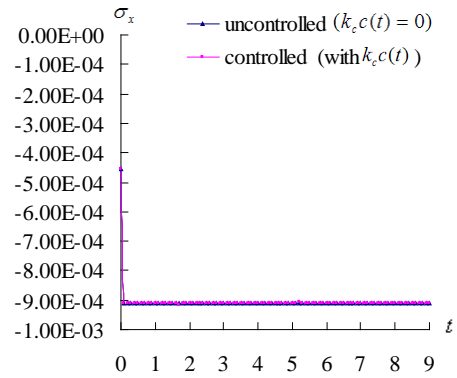


FIG. 3b  $\sigma_x$  vs.  $t$  for  $a/h^* = 20$

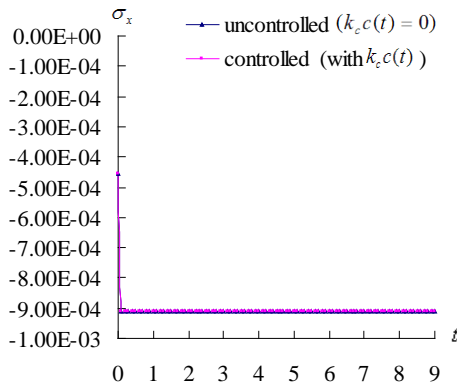


FIG. 3c  $\sigma_x$  vs.  $t$  for  $a/h^* = 100$

FIG. 3  $\sigma_x$  vs.  $t$  for  $a/h^* = 5, 20$  AND  $100$  WITHOUT SHEAR

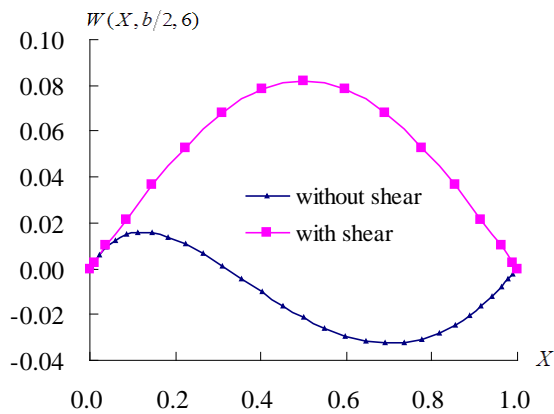


FIG. 4a  $W(X, b/2, 6)$  vs.  $X$  for  $a/h^* = 5$

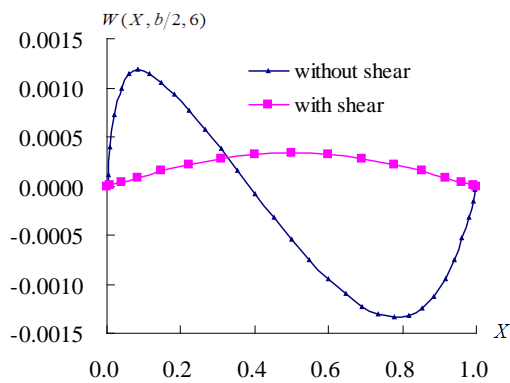


FIG. 4b  $W(X, b/2, 6)$  vs.  $X$  for  $a/h^* = 20$

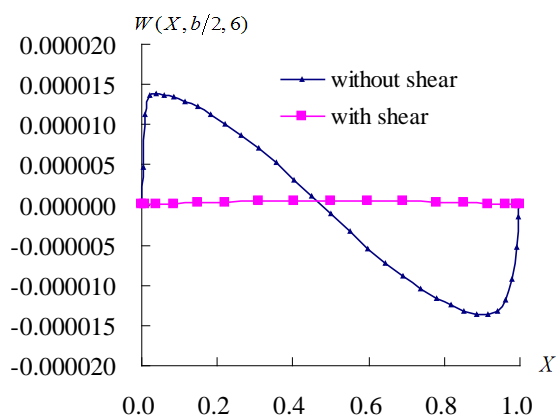


FIG. 4c  $W(X, b/2, 6)$  vs.  $X$  for  $a/h^* = 100$

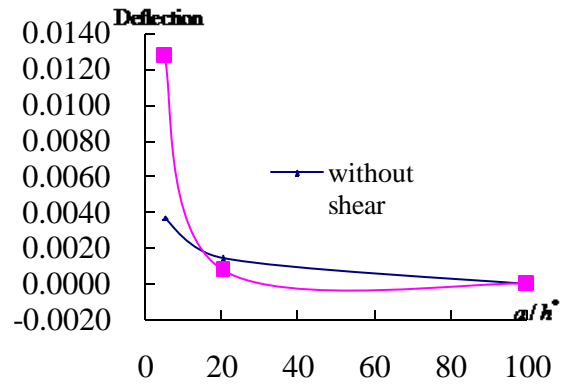


FIG. 4d CENTER DEFLECTION vs.  $a/h^*$  AT  $t = 6$  sec  
FIG. 4 COMPARED  $W(X, b/2, 6)$  vs.  $X$  for  $a/h^* = 5, 20$  AND  $100$

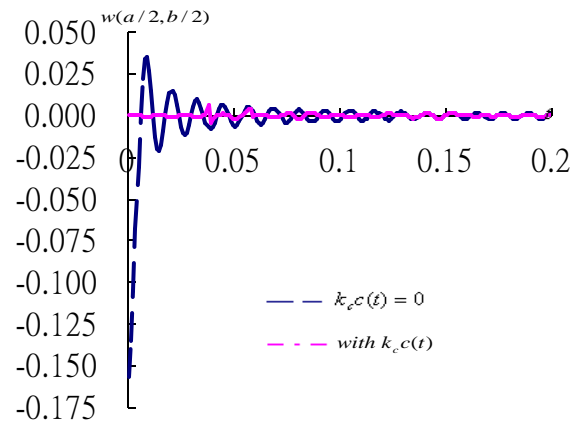


FIG. 5a TRANSIENT VALUE OF  $w(a/2, b/2)$  vs.  $t$  FOR  $a/h^* = 5$

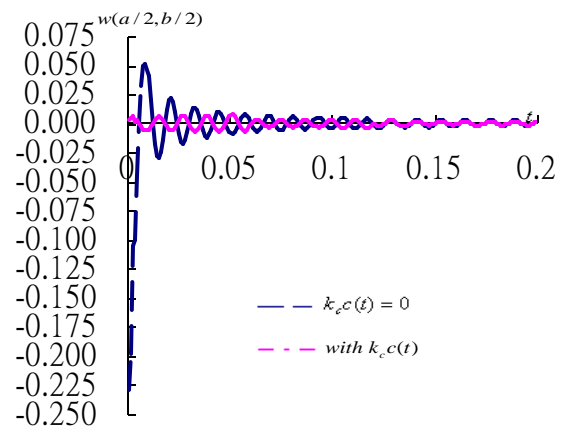
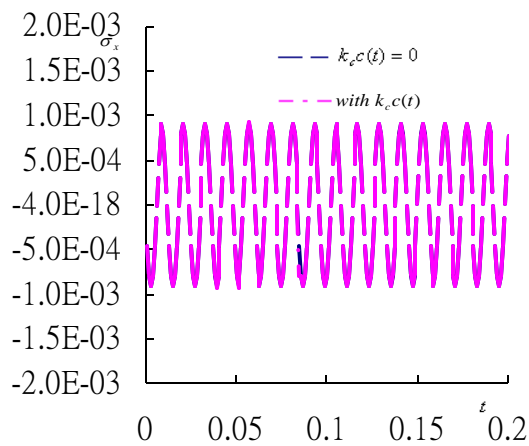
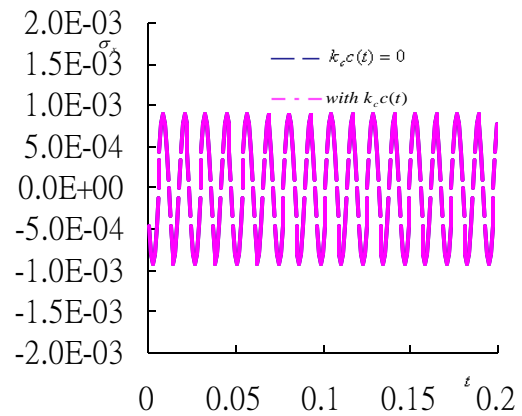


FIG. 5b TRANSIENT VALUE OF  $w(a/2, b/2)$  vs.  $t$  for  $a/h^* = 100$

FIG. 5 TRANSIENT VALUE OF  $w(a/2, b/2)$  vs.  $t$  for  $a/h^* = 5$  AND  $100$  WITHOUT SHEAR

FIG. 6a TRANSIENT VALUE OF  $\sigma_x$  vs.  $t$  FOR  $a/h^* = 5$ FIG. 6b TRANSIENT VALUE OF  $\sigma_x$  vs.  $t$  FOR

$$a/h^* = 100$$

FIG. 6 TRANSIENT VALUE OF  $\sigma_x$  vs.  $t$  FOR  $a/h^* = 5$   
AND 100 WITHOUT SHEAR

TABLE 1 TEMPERATURE-DEPENDENT COEFFICIENT OF CONSTITUENT FGM MATERIALS

Materials	$P_i$	$P_0$	$P_{-1}$	$P_1$	$P_2$	$P_3$
SUS304	$E_1 (P_a)$	201.04E09	0	3.079E-04	-6.534E-07	0
	$\nu_1$	0.3262	0	-2.002E-04	3.797E-07	0
	$\rho_1 (Kg/m^3)$	8166	0	0	0	0
	$\alpha_1 (^\circ K^{-1})$	12.33E-06	0	8.086E-04	0	0
	$\kappa_1 (W/m^\circ K)$	15.379	0	0	0	0
	$C_{v1} (J/Kg^\circ K)$	496.56	0	-1.151E-03	1.636E-06	-5.863E-10
$Si_3N_4$	$E_2 (P_a)$	348.43E09	0	-3.70E-04	2.16E-07	-8.946E-11
	$\nu_2$	0.24	0	0	0	0
	$\rho_2 (Kg/m^3)$	2370	0	0	0	0
	$\alpha_2 (^\circ K^{-1})$	5.8723E-06	0	9.095E-04	0	0
	$\kappa_2 (W/m^\circ K)$	13.723	0	0	0	0
	$C_{v2} (J/Kg^\circ K)$	555.11	0	1.016E-03	2.92E-07	-1.67E-10

TABLE 2 DYNAMIC CONVERGENCE OF TERFENOL-D FGM PLATE WITHOUT SHEAR

$a/h^*$	GDQ method	$w(a/2, b/2)$ (unit mm)		
	$N \times M$	$a/b = 0.5$	$a/b = 1.0$	$a/b = 2.0$
100	$17 \times 17$	-0.634165E-04	-0.905412E-04	-0.175673E-03
	$21 \times 21$	-0.563660E-04	-0.796810E-04	-0.164627E-03
	$25 \times 25$	-0.518107E-04	-0.735170E-04	-0.160291E-03
	$29 \times 29$	-0.494592E-04	-0.709745E-04	-0.158750E-03
	$33 \times 33$	-0.483320E-04	-0.701375E-04	-0.158280E-03
50	$17 \times 17$	-0.202713E-03	-0.283312E-03	-0.585536E-03
	$21 \times 21$	-0.191526E-03	-0.271641E-03	-0.579210E-03
	$25 \times 25$	-0.187638E-03	-0.269502E-03	-0.577589E-03
	$29 \times 29$	-0.186386E-03	-0.269140E-03	-0.577392E-03
	$33 \times 33$	-0.186190E-03	-0.269511E-03	-0.577425E-03
20	$17 \times 17$	-0.999116E-03	-0.131836E-02	-0.205093E-02
	$21 \times 21$	-0.987938E-03	-0.131330E-02	-0.205050E-02
	$25 \times 25$	-0.981546E-03	-0.131383E-02	-0.205044E-02
	$29 \times 29$	-0.990436E-03	-0.131042E-02	-0.205094E-02
	$33 \times 33$	-0.990760E-03	-0.130088E-02	-0.205010E-02
10	$17 \times 17$	-0.241265E-02	-0.283326E-02	-0.293830E-02
	$21 \times 21$	-0.241384E-02	-0.283509E-02	-0.293858E-02
	$25 \times 25$	-0.241151E-02	-0.283366E-02	-0.293846E-02
	$29 \times 29$	-0.241165E-02	-0.283456E-02	-0.293842E-02
	$33 \times 33$	-0.240542E-02	-0.283410E-02	-0.294010E-02
5	$17 \times 17$	-0.340208E-02	-0.343168E-02	-0.232384E-02
	$21 \times 21$	-0.340130E-02	-0.342991E-02	-0.232400E-02
	$25 \times 25$	-0.340583E-02	-0.343044E-02	-0.232387E-02
	$29 \times 29$	-0.340369E-02	-0.343035E-02	-0.232335E-02
	$33 \times 33$	-0.339761E-02	-0.343055E-02	-0.232343E-02

TABLE 3  $\gamma$  AND  $\omega_{11}$  OF TERFENOL-D FGM PLATE WITHOUT SHEAR

$a/h^*$	$t = 0.001$ s		$t = 1$ s		$t = 2$ s	
	$\gamma$	$\omega_{11}$	$\gamma$	$\omega_{11}$	$\gamma$	$\omega_{11}$
100	523.599	0.569829E-02	1.57080	0.141884E-01	0.785398	0.569830E-02
20	523.599	0.567729E-02	1.57080	0.151387E-01	0.785398	0.567730E-02
5	523.599	0.196036E-01	1.57080	0.136018E-01	0.785400	0.538113E-02
$a/h^*$	$t = 3$ s		$t = 4$ s		$t = 5$ s	
	$\gamma$	$\omega_{11}$	$\gamma$	$\omega_{11}$	$\gamma$	$\omega_{11}$
100	0.523599	0.569830E-02	0.392699	0.569830E-02	0.314159	0.569829E-02
20	0.523599	0.567730E-02	0.392699	0.567730E-02	0.314159	0.567729E-02
5	0.523599	0.538112E-02	0.392701	0.538113E-02	0.314161	0.538113E-02
$a/h^*$	$t = 6$ s		$t = 7$ s		$t = 8$ s	
	$\gamma$	$\omega_{11}$	$\gamma$	$\omega_{11}$	$\gamma$	$\omega_{11}$
100	0.261799	0.569830E-02	0.224400	0.569830E-02	0.196349	0.569830E-02
20	0.261799	0.567730E-02	0.224400	0.567730E-02	0.196349	0.567730E-02
5	0.261800	0.538113E-02	0.224401	0.538112E-02	0.196351	0.538113E-02
$a/h^*$	$t = 9$ s					
	$\gamma$	$\omega_{11}$				
100	0.174532	0.569830E-02				
20	0.174532	0.567730E-02				
5	0.174534	0.538113E-02				

TABLE 4  $k_c c(t)$  VERSUS  $t$  FOR  $R_n = 1$  WITHOUT SHEAR

$a/h^*$	$k_c c(t)$					
5	$t = 0.001 \sim 0.1$	$t = 0.2 \sim 0.4$	$t = 0.5 \sim 0.9$	$t = 1.0$	$t = 1.1 \sim 1.4$	$t = 1.5$
	$-10^9$	$-3 \times 10^6$	$-10^7$	$-10^2$	$-10^9$	$-10^7$
	$t = 1.6 \sim 3.3$	$t = 3.4 \sim 3.9$	$t = 4.0 \sim 4.6$	$t = 4.7 \sim 9.0$		
	$-10^9$	$-2 \times 10^9$	$10^9$	$1.56 \times 10^9$		
20	$t = 0.001$	$t = 0.1$	$t = 0.2$	$t = 0.3 \sim 0.9$	$t = 1.0$	$t = 1.1$
	$-10^9$	$10^6$	$-10^9$	$-10^5$	$-10^2$	$-10^9$
	$t = 1.4 \sim 3.2$	$t = 3.3 \sim 3.5$	$t = 3.6$	$t = 3.7 \sim 5.2$	$t = 5.3 \sim 5.4$	
	$10^9$	$10^5$	$10^2$	$-10^9$	$10^5$	
	$t = 5.5 \sim 7.5$	$t = 7.6 \sim 9.0$				
	$-10^7$	$10^9$				
100	$t = 0.001$	$t = 0.1 \sim 1.5$	$t = 1.6 \sim 1.9$	$t = 2.0$	$t = 2.1 \sim 2.3$	$t = 2.4$
	$-10^9$	$10^2$	$-10^9$	$-10^2$	$10^7$	$-10^9$
	$t = 2.5 \sim 9.0$					